Using hybrid optimization algorithms for very-large graph problems and for small real-time problems

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The opinions expressed in this talk are those of the authors and do not necessarily represent the views of the FCC or any of its staff.

What to do when state-of-the-art software does not solve the problem?

Three Important and Difficult Problems:

- The Federal Communications Commission Incentive Auction
  - Why the problem is important
  - Approach to solving the problems
  - Impact to government policy making

- Assigning Channel Assignments to Radios on the Battlefield
  - Similar to the FCC problem, but now there is a dynamic aspect to the problem

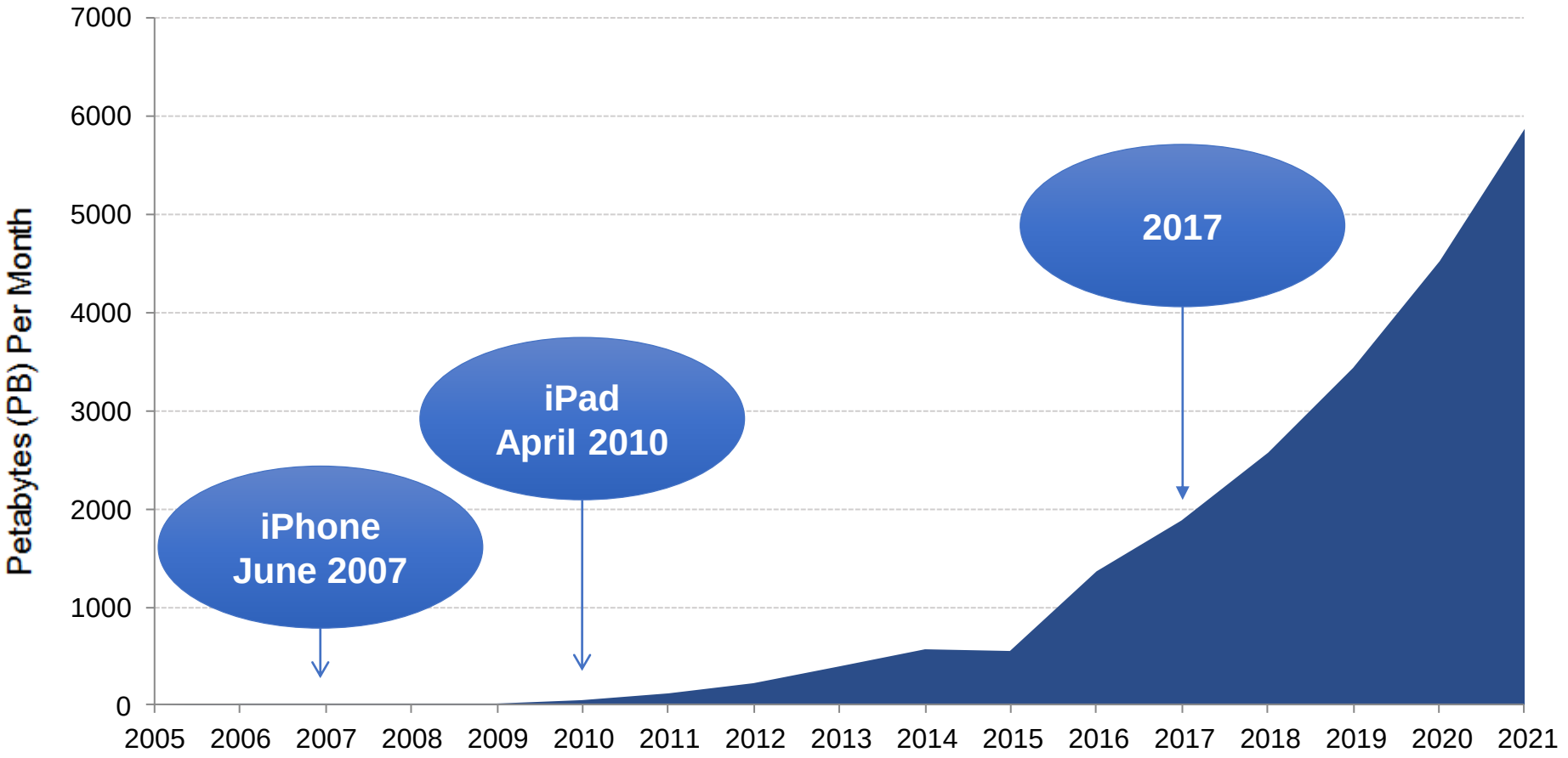
- Getting near-optimal solutions to routing problems for the App Economy
  - The future of optimization

The FCC: UNLOCKING THE BEACHFRONT, Using Operations Research To Repurpose Spectrum

- The Federal Communications Commission (FCC) recently completed the world's first two-sided spectrum auction
- The auction was one of the most successful in FCC history
  - Repurposed 84 MHZ of spectrum
  - Generated revenue of nearly $20 billion
- Optimization was essential to the design and implementation of the auction

Introduction

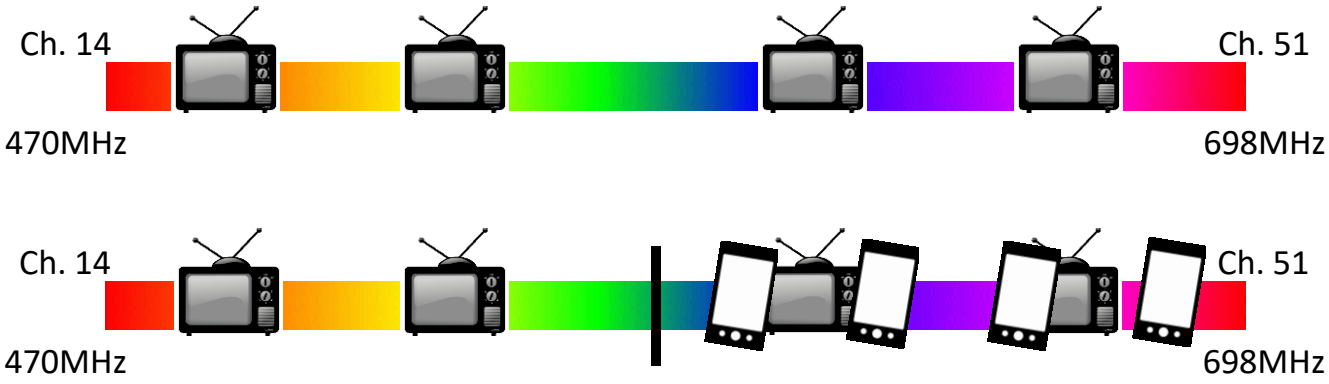
U.S. Mobile Data Traffic Growth Forecast



*Cisco VNI Forecasts; FCC Analysis

Introduction

What is the broadcast Incentive Auction?



- Repack the television stations onto fewer channels
- Auction newly freed spectrum to wireless bidders

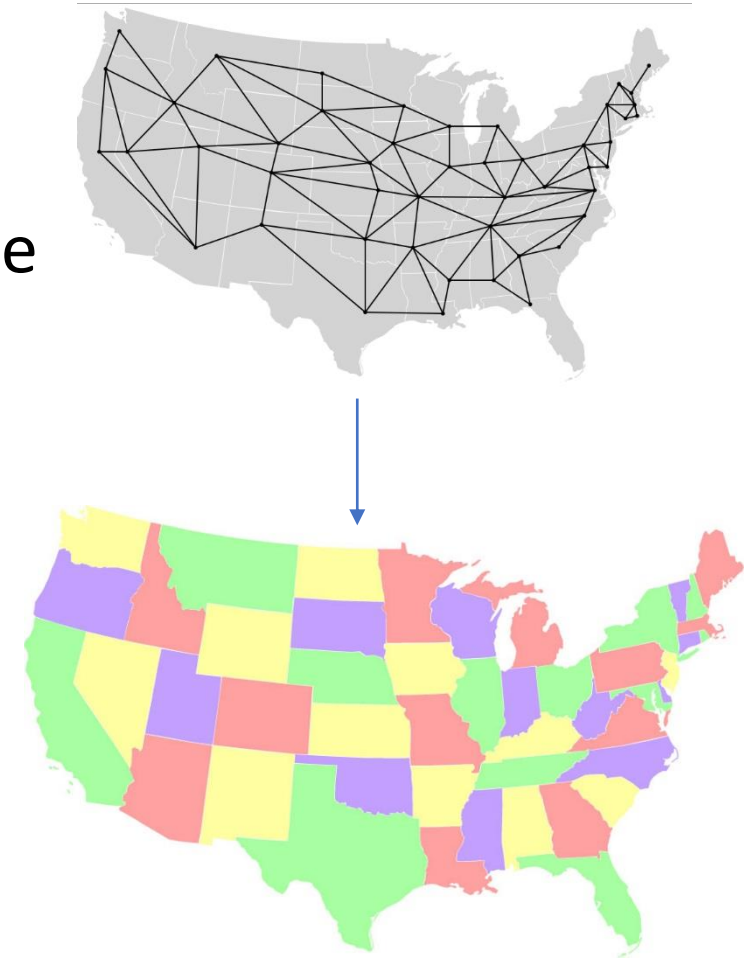
Repacking the stations is the equivalent of a graph coloring problem (with side constraints and objectives!)

Other Graph Coloring Problems

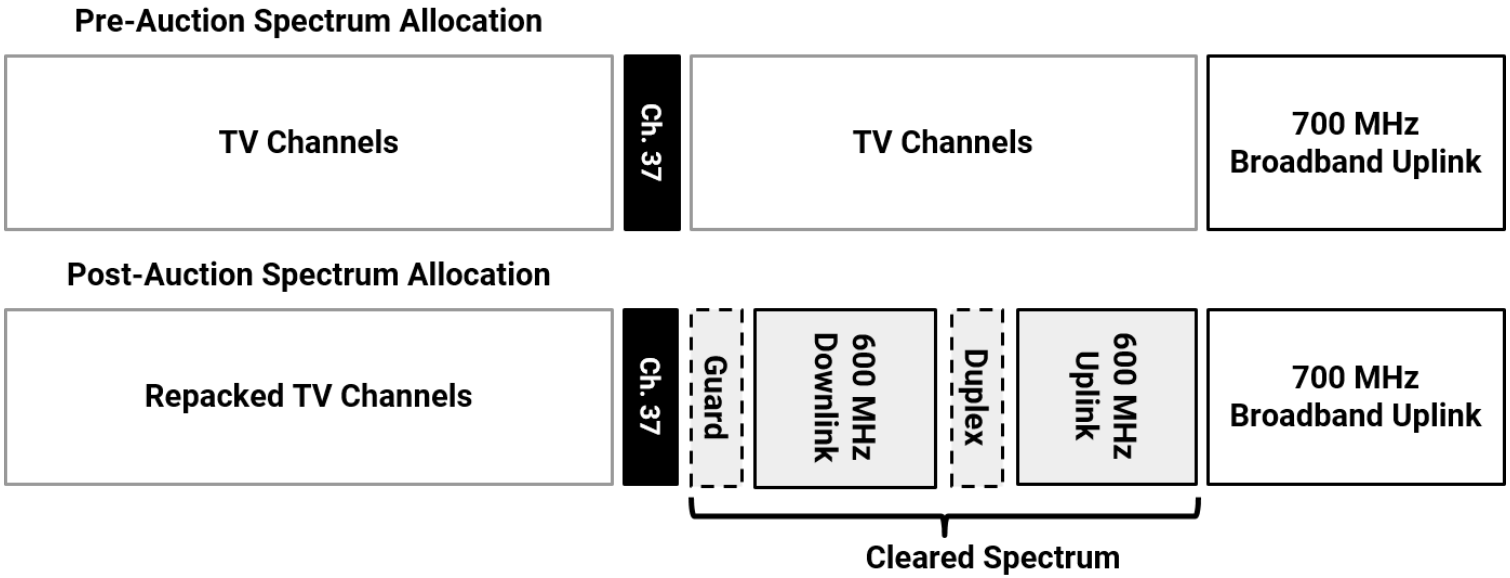
- Map coloring

- Creating a schedule or time table
  - Crew Scheduling
  - Round Robin Sport Scheduling
  - Security Camera Scheduling
  - Scheduling Software Updates

- Air Traffic Flow Management



Pre-Auction vs. Post-Auction 600 MHz Band



The Reallocation Problem

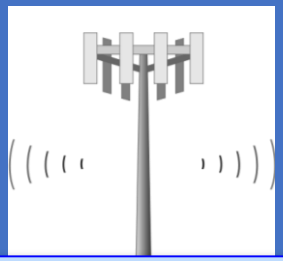
- Simple market solutions insufficient
  - International coordination of frequency uses.
  - Different license rights for broadcast versus broadband.
  - Need to move non-sellers and provide guard bands.
  - Computational challenges in reassigning stations.

- *"Middle Class Tax Relief and Job Creation Act of 2012"* gave FCC the right to retune holdouts, turning stations on different frequencies into substitutes. Thus, Broadcasters have the right to continue broadcasting on some channel with *no increase in interference,* but *not* to remain on the current channel!

Broadband Incentive Auction: Key Components

Broadcasters

- Offer to relinquish spectrum usage rights



Mobile Broadband Providers

- Offer to purchase spectrum licenses

Reverse Auction

Integration

Forward Auction

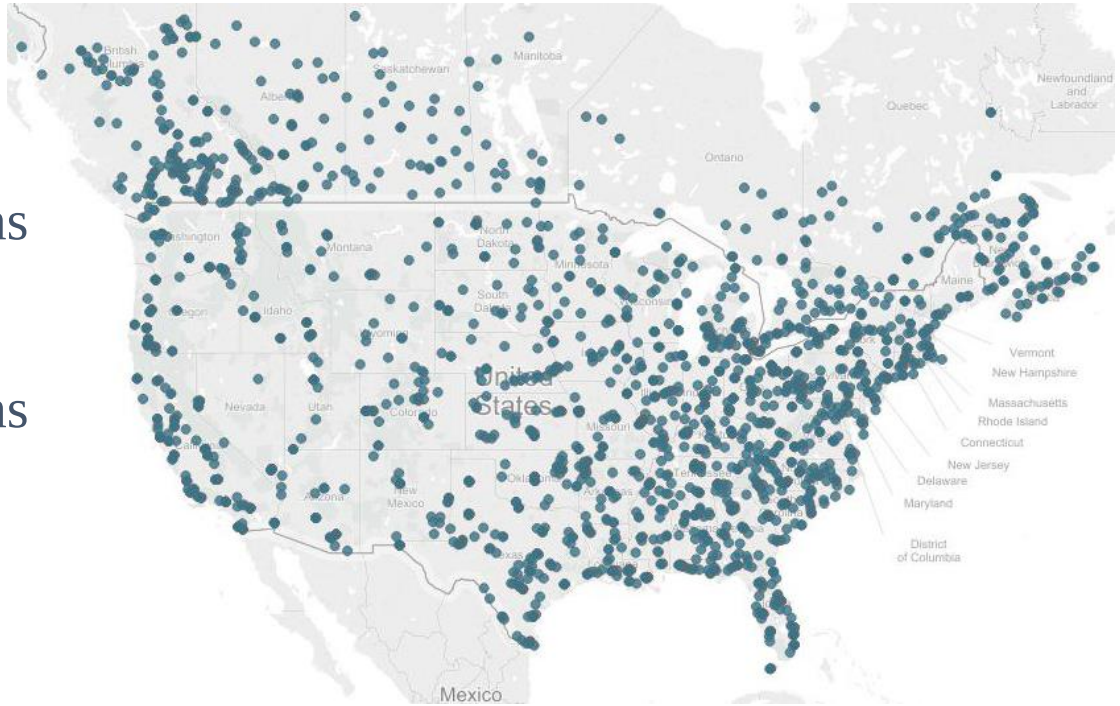
The Auction Framework

- Stations voluntary participate, accept bids (payments) to vacate their channel
  - Cease operation and go off the air
  - Move to a lower TV band
- If more stations bid than are needed to clear the spectrum, the bid value decreases until a station no longer accepts a bid and gets placed back on air
- Auction closes once no more stations can be individually packed into the new TV bands

The Reallocation Problem

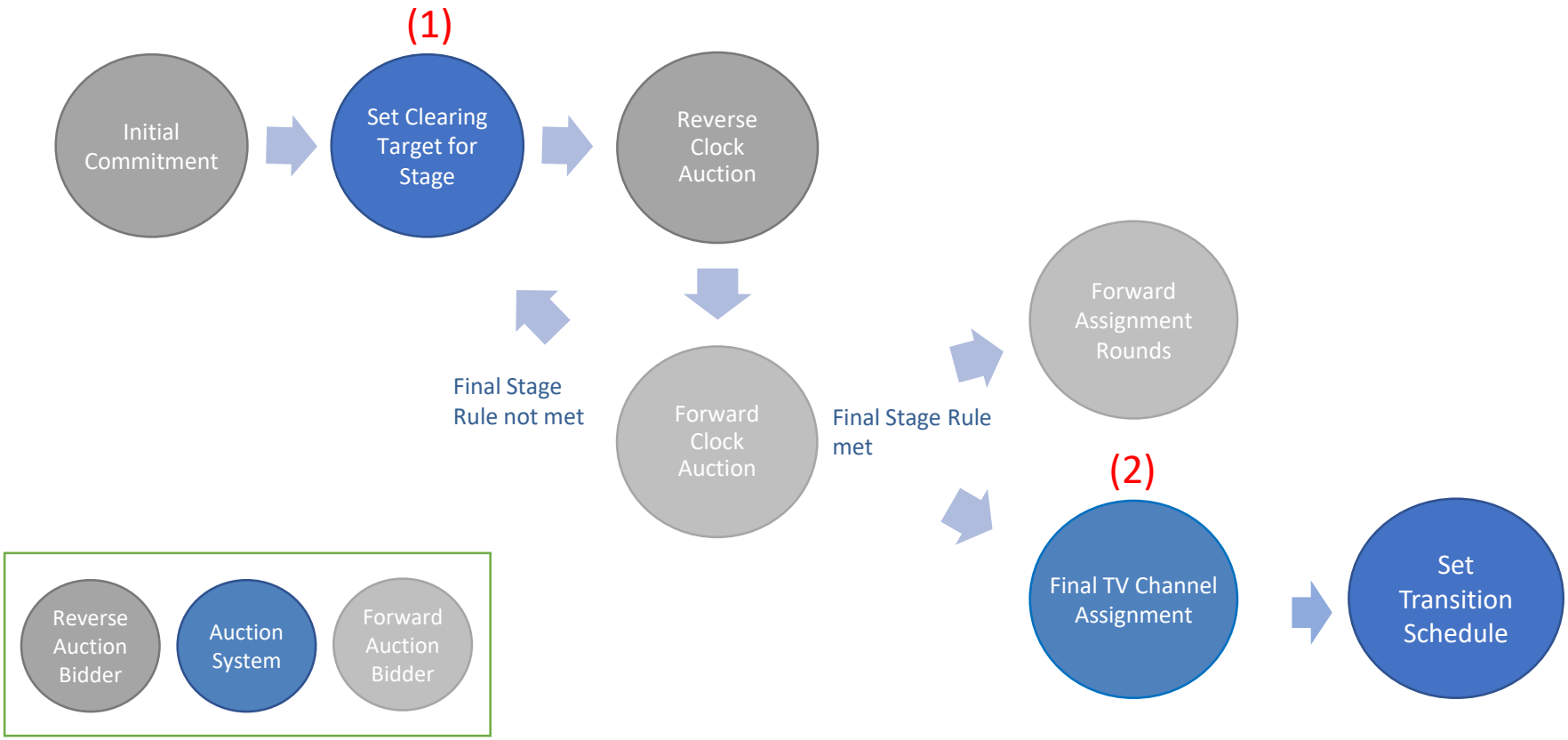
U.S. AND CANADIAN STATIONS

- 2197 US Stations
  - 1709 UHF Stations
- 793 Canadian Stations
  - 348 UHF Stations



Many Optimizations within the Auction Framework…

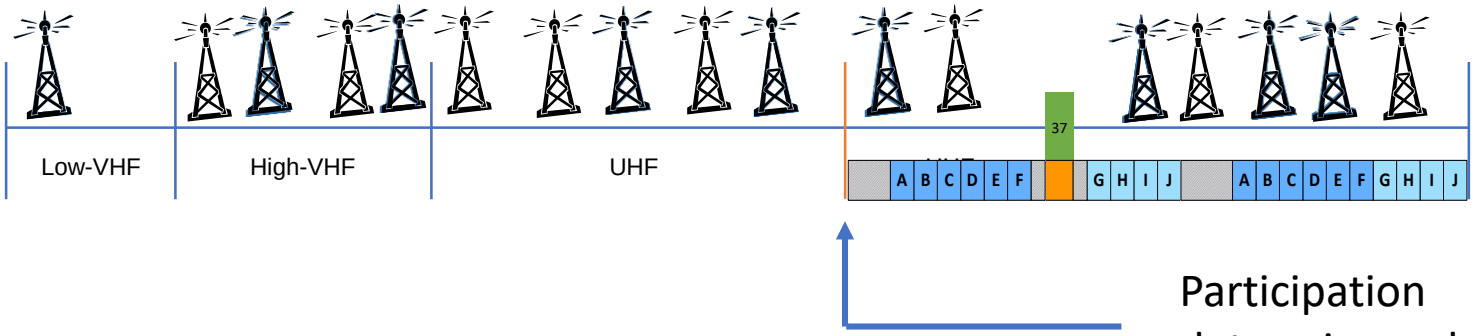
The two highlighted required optimizations solutions really quickly or provable really good; All used HYBRID approaches.



Optimization Descriptions

Clearing Target Optimization

- Determines the maximum amount of spectrum to be auctioned.
  - Allows stations to be assigned in wireless band to prevent the most congested market from determining the amount of spectrum available

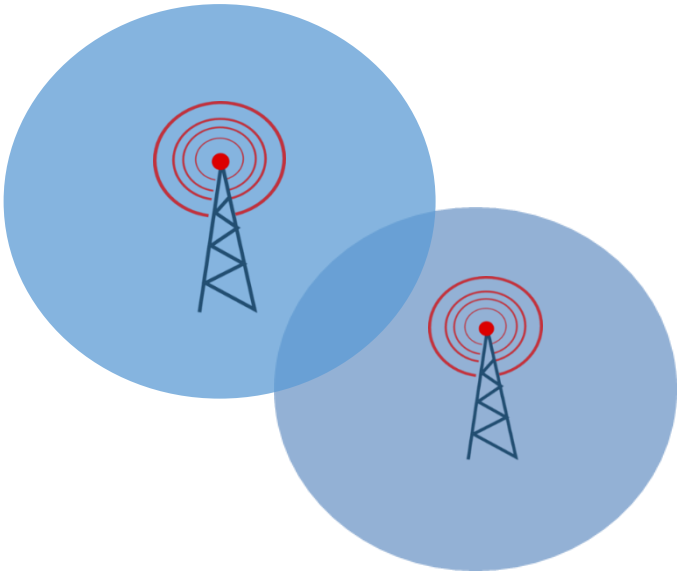


Final Channel Assignment

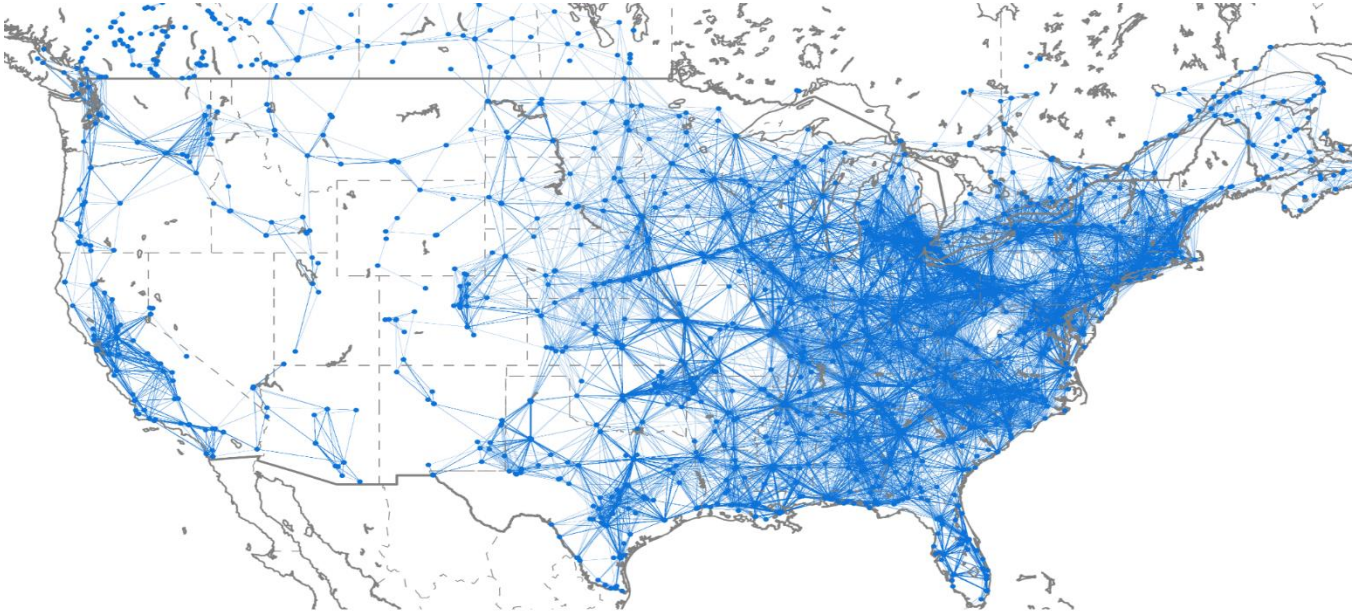
- After the auction is complete, determines what channels the stations will use.
  - Assign stations to their current channel when possible and minimize new interference

The Major Modeling Issue: How to Model Interference Protections

- New station assignments cannot interfere more than 0.5% of another station's current population
- Potential pairwise interference must be studied for stations assigned to the same channel or to adjacent channels
- 2,694,283 pairwise interference restrictions
- Near (but not exact) symmetric restrictions across different channels because of the adjacent channel requirements



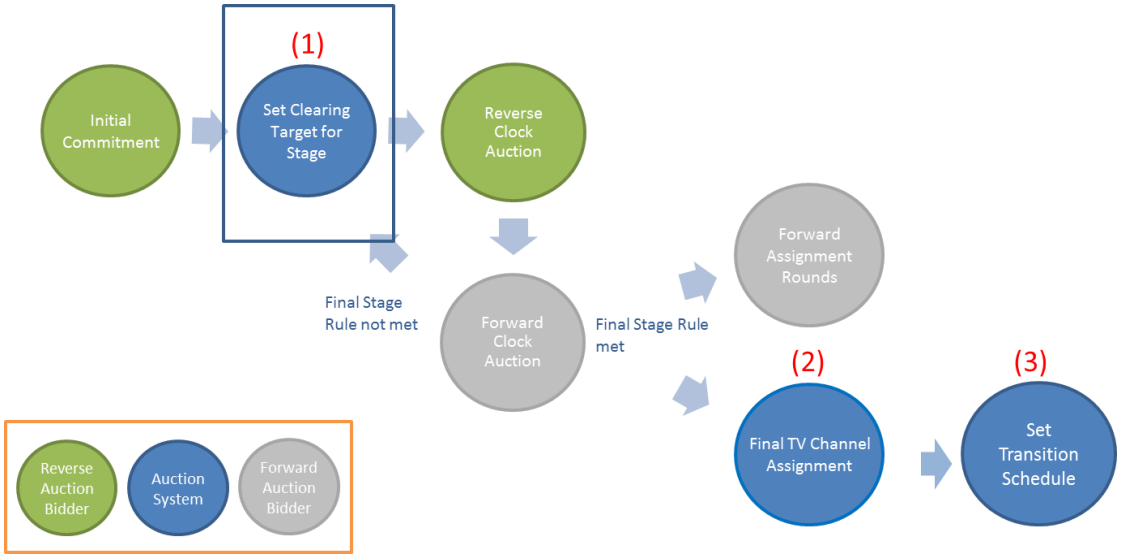
Protecting Stations From Interference



This created over 2.5 MILLION pairwise interference protections to ensure that stations still reach their viewers

Clearing Target Optimizations:

Critically Important Model



- Determined where the auction started
- Potentially assigned stations to wireless bands
- Poor performance here could result in BILLIONS $ lost
  - Less revenue in forward auction due to less spectrum
  - Lower prices for stations due to less competition
  - Less benefit to economy due to inefficient spectrum use

Goals of Clearing Target Determination

- Goal was to repurpose as much spectrum as possible on a *nationwide* basis, this presented the FCC with a choice:
- Either:
  a. limit the amount of spectrum repurposed in every market to the amount made available in the most constrained market (Lowest Common Denominator) or
  b. allow some TV stations to be assigned channels in the new wireless services band if broadcaster clearing in a market was not sufficient.

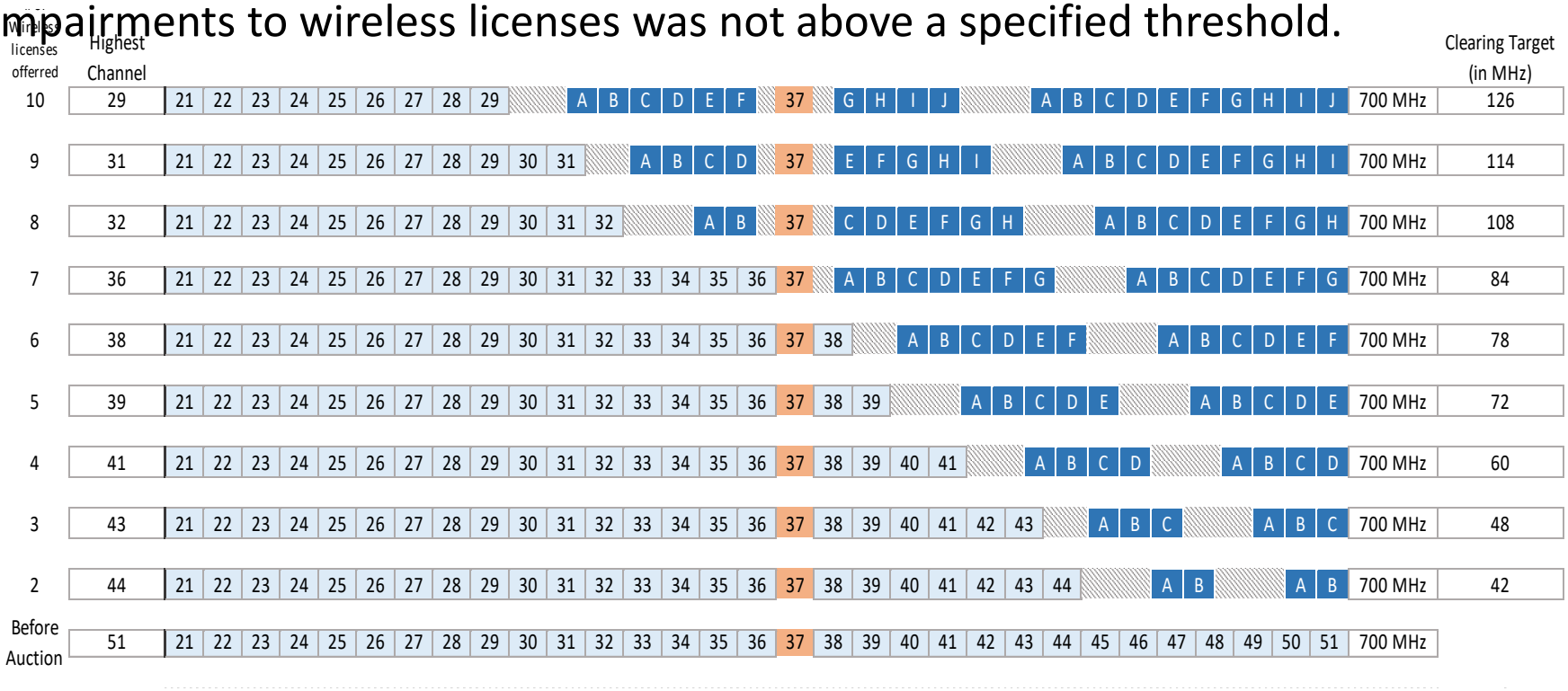
The FCC chose to allow TV stations to be assigned in the new wireless services band.

Optimization determined:
The Stations to be put in the Wireless Band and the Channel Assignments for those Station.

Clearing Target Optimization: Band Plans

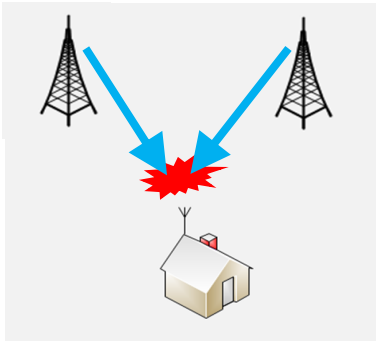
The FCC proposed a collection of band plans, that would protect TV stations from interference with wireless through Guard Bands (the gray areas on the chart below).

The Channel Assignment chose the highest clearing target as long as the impairments to wireless licenses was not above a specified threshold.



Clearing Target Optimizations

What makes it difficult

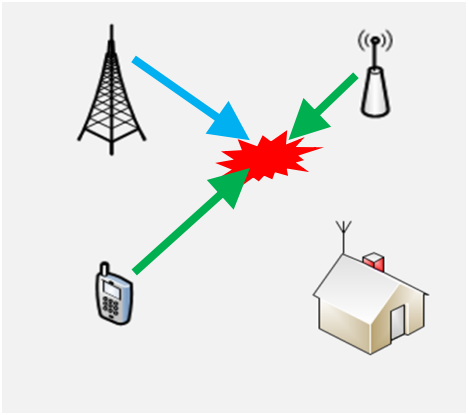


TV 2 TV Interference

- Over 2 million pairwise protections

Inter Service Interference

- Protecting interference between different services (Wireless vs TV)
- Over 6 million additional protections!!
- Protects on an *aggregate, county* level



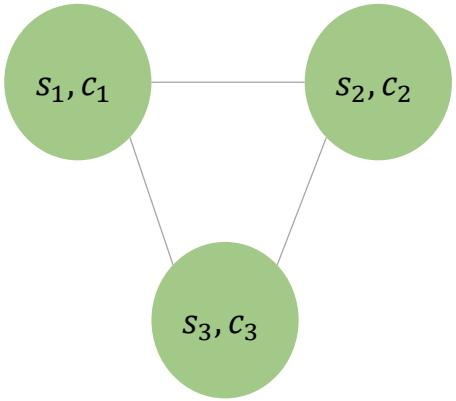
How the Clearing Target Worked:

- All optimizations included:
  - The TV Repacking constraints
  - Inter-service interference constraints
- Canadian constraints were added to allow a larger guard band between TV and Wireless
- Other constraints assured that US TV stations who participated in the auction would have as much flexibility in bidding as possible
- *Primary Objective for Clearing Target Optimization*: Determine the Minimum sum of impaired weighted pops across all licenses in both the US and Canada
- *Secondary Objective:* Determine the maximum number of unimpaired licenses

Formulation Matters

Analyze the constraints to identify more efficient formulations

- The millions of pairwise constraints could be consolidated into better clique constraints



$$x_1 + x_2 \leq 1$$
$$x_2 + x_3 \leq 1$$
$$x_1 + x_3 \leq 1$$

or $x_1 + x_2 + x_3 \leq 1$

In addition, we ran different tests to determine the best set of cliques;
2.5 million pairwise constraints reduced to **220,000** clique constraints
For the inter-service interference constraints, **>6million** to about **680,000**

Exploit Problem Structure

Identify an alternative formulation or decomposition

For the clearing target problem, we were identifying stations to be assigned to the wireless band.

We decomposed the problem into a feasible packing problem in the TV Band and then a best assignment problem in the wireless band.

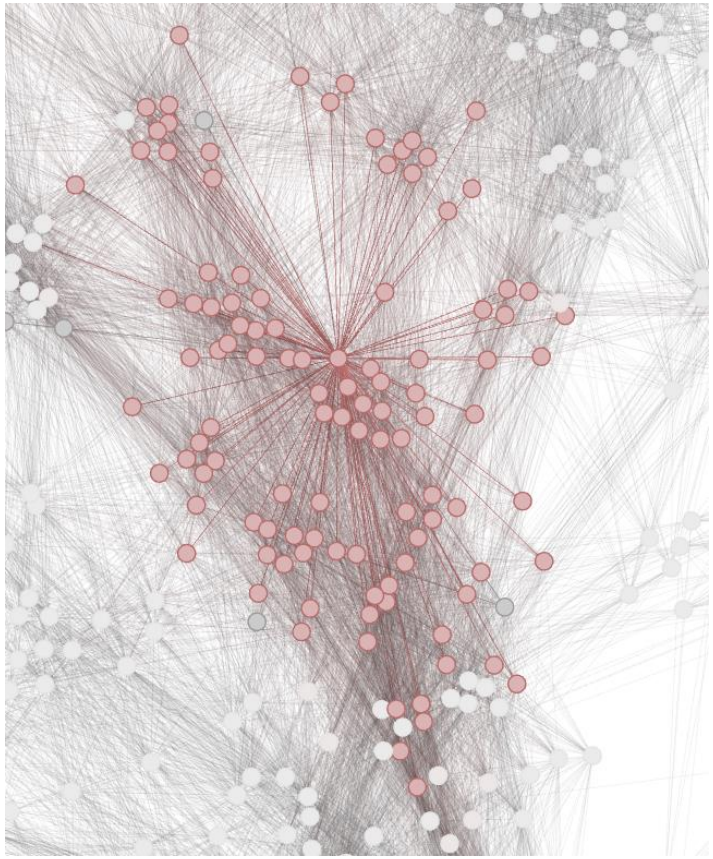
Assignments to the wireless band would only be in congested areas so we could also separate geographic regions.

We approached the problem as a Logic-based Benders Decomposition where the “cuts” generated told the Master Problem (wireless assignments) about the TV band solution possibilities.

Build your own heuristics

Gurobi's heuristics are great!!!! But they approach the problem from the formulation provided

- Use your knowledge of what you are modeling to identify other potential heuristics
- We did local optimizations by geography to improve regions of the country
- Similarly, locally optimize over *specific* stations or channels



Take advantage of the features of the solver

Our goal was near optimal solutions in a reasonable amount of time.

- We used Gurobi 6.5 and created our own distributed solver.
  - Some solvers used Gurobi with different parameter configurations
  - Some solvers used Gurobi with our custom heuristics

- Using callbacks, we communicated solutions between solvers.

- Where appropriate, lazy constraints can make the problem much smaller and easier to solve

Another Difficult Optimization Problem: FINAL CHANNEL ASSIGNMENT

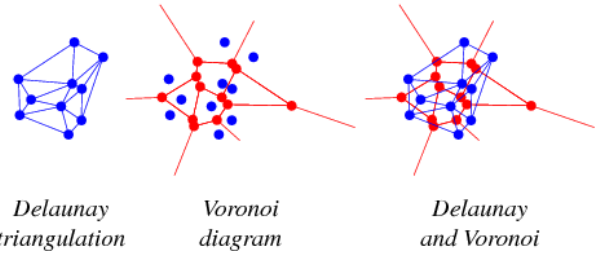
- Once the auction ended, the Final Channel Assignment determined the channel assignments for all stations remaining on the air.

- FCC chose to optimize the following goals
  - Assign stations to their current channel when possible
  - Minimize new aggregate interference
  - Avoid moving stations whose move would be exceptionally difficult and/or expensive

Final Channel Optimization Problems

HOW WE OBTAINED GOOD SOLUTIONS IN REASONABLE TIMES:

- Clique constraints
  - 6 Million constraints → 500,000 constraints
- Custom heuristic: used a distance-based metric via Delaunay triangulation. (Used lat./long information)



- Benders decomposition:
  - Master problem: choose stations to remain on current channel)
  - Sub-problems: (Find where assignment not possible and add cuts to Master)
- Again Used a Distributed Computing Environment

For all optimizations: Used distributed solver technology

Our goal was near optimal solutions in a reasonable amount of time

We used Gurobi 6.5 and created our own distributed solver

- Some solvers used Gurobi with different parameter configurations
- Some solvers used Gurobi with our custom heuristics

Using callbacks, we communicated solutions between solvers

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Results of Hybrid Optimization Approach

- We solved each Clearing Target to proven optimality!
  - Allowed the auction to start at the highest clearing target (126 MHz)
  - Starting at a high clearing target encouraged stations to participate
  - Ability to get good solutions helped policy analysis

- In Final Channel Assignment, far less stations needed to move than industry expected
  - Nearly 1700 stations remained on their original channel
  - Maximum aggregate interference for any station was 1.1%
  - Only 12 stations received more than 1% aggregate interference

Results of FCC Auction

Auction Results:

- Final band plan had 84 MHz of completely clear spectrum
- Wireless providers paid almost $20 Billion for the spectrum
- Broadcasters received over $12 Billion
- Over $7 Billion to the US Treasury

Optimization contributions:

- We solved each Clearing Target to proven optimality!
- In Final Channel Assignment, far less stations needed to move than industry expected

And…

- Optimization team helped design the Transition Schedule for the remaining stations!

Lessons Learned From FCC Application

- Understand your problem
  - What are you modeling?
  - What results are your stakeholders looking for?
- Be creative in solving your problem
- In policy modeling, confidence building is essential and takes time and effort…need to give insights rather than answers
- When the problem seems unsolvable:
  - Often a smaller version of the problem is solvable so exploit this fact for heuristic search and for improving other bound (using multiple optimizations to solve a single optimization)
  - *Exploit the inherent structure.*

A related Problem:
How to assign channels to radios on the battlefield

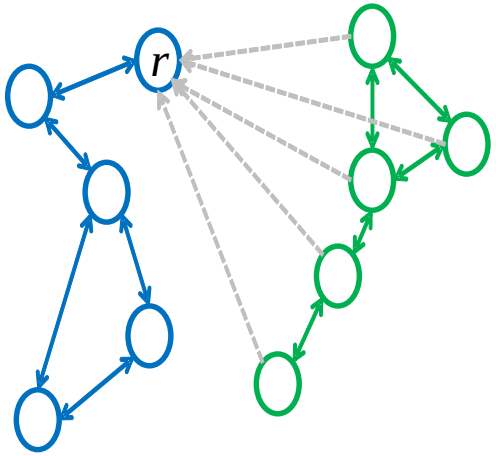
We consider three challenges:

- Minimizing the number of required channels
  - *Minimum-order channel assignment problem* (***MO-CAP***)
- Minimizing interference, given a fixed number of channels
  - *Minimum-interference channel assignment problem* (***MI-CAP***)
- Minimizing the number of channel changes over time
  - *Minimum-cost channel assignment problem over time* (***MC-CAP-T***)

Goal: To solve realistic, full-sized instances of various *channel assignment problems* (*CAPs*) in a reasonable amount of time

Model of MANET Operations

- A military *unit* (e.g., a company or battalion headquarters) uses a MANET to communicate on an assigned channel. (Units indicated by blue and green.)
- Each unit's MANET may comprise up to 30 radios (indicated by circles)
- Only radios within a particular unit may communicate (indicated by solid arrows); there is no MANET *backhaul network* between units (in reality, this is provided via other transmission systems)
- All other radios assigned to a different unit but operating on the same channel provide co-channel interference (gray dashed arrows)
- Interference is cumulative at a receiver (e.g., radio *r*)



Solution Approach

- Develop an integer programming model of MANET operations that captures the most important aspects of communications
  - Mobile units on rough terrain
  - Cumulative co-channel interference
- Generate realistic input data based on high-fidelity combat scenarios
  - Derived from Defense Planning Scenarios
  - Scenarios instantiated and radios simulated using AGI Systems Toolkit (STK)[1]
  - Radio propagation simulated using the Terrain Integrated Rough Earth Model (TIREM)[2]
- Develop several formulations of each CAP
- Use heuristics, integer optimization, and constraint programming to solve each of the three problem

How to overcome computational difficulties?

- Substitute pair-wise interference constraints for aggregate interference constraints.
- For every integer solution found in the process, check if aggregate interference violated…
  - Whenever a "feasible" integer solution is found, callback checks the aggregate feasibility (in polynomial time)
  - Higher-order packing constraints dynamically added as needed to reduce / eliminate infeasibility:

$$\sum_{u \in S} X_u^c \leq |S| - 1$$

- Strengthen pairwise formulation through clique constraint
- With these "tricks", CPLEX gets good solutions BUT… lower bound still bad
- SOLUTION: Use constraint programming to get better lower bound

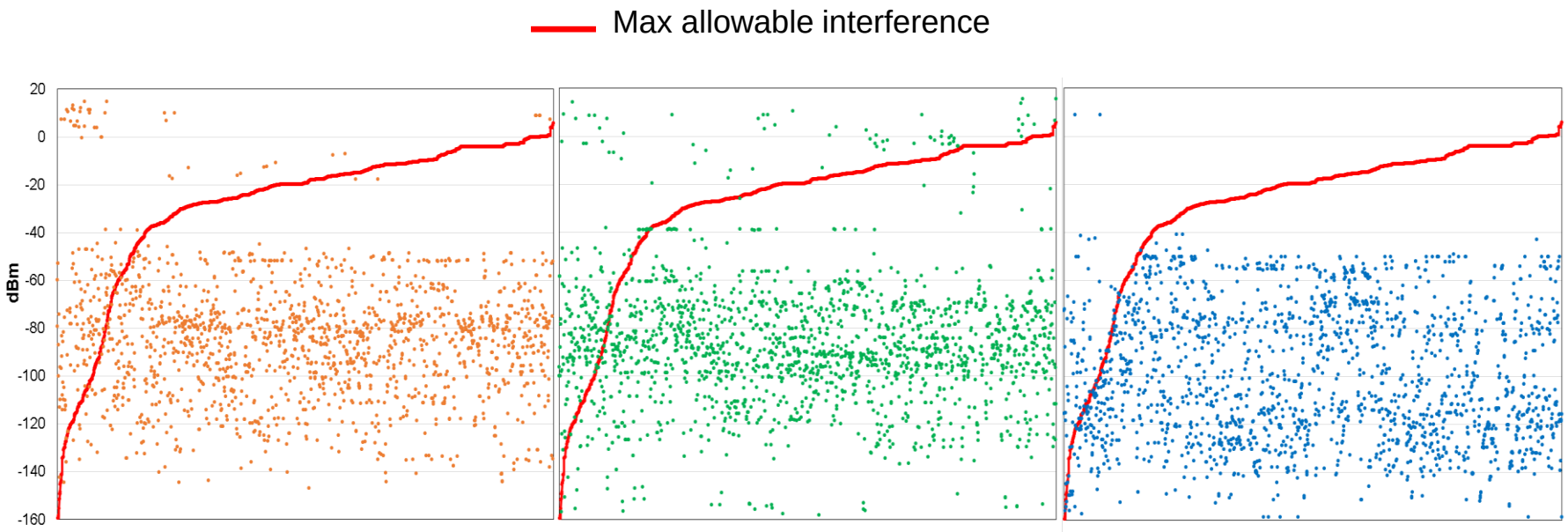
What if there is not enough spectrum?

- Again, we formulate as an IP, considering only pairwise interference to capture a substantial portion of interference
- We attempt to minimize the number of pairwise constraint violations
- We implement using Python and CPLEX

RESULTS:

- Tested optimization versus constraint programming for same formulation
- We run each method with varying levels of *channel availability*
- We calculate *network availability* to estimate the operational impact upon the ability to use each MANET
  - Percentage of radios able to communicate with *network control radio*
- Overall, Constraint Programming provides superior performance in considerably less time, and provides good bounds

Results



Clustering
Total excess. interf: 22.88 dBm
radios inoperable: 174

IP
Total excess. interf: 25.52 dBm
radios inoperable: 224

CP
Total excess. interf: 17.53 dBm
radios inoperable: 96

FINAL APPLICATION: Using Optimization in the App Economy
The Dynamic Pickup & Delivery Problem for Restaurant Delivery Services

Given:

- Orders that need to be picked up from restaurants and delivered
- Target pickup times for the orders
- Target service levels for getting food to diners
- Geographically distributed couriers who may already have assignments

Determine:

- Which driver gets each order
- Which route each driver should take
- When to dispatch each order, given the current plan

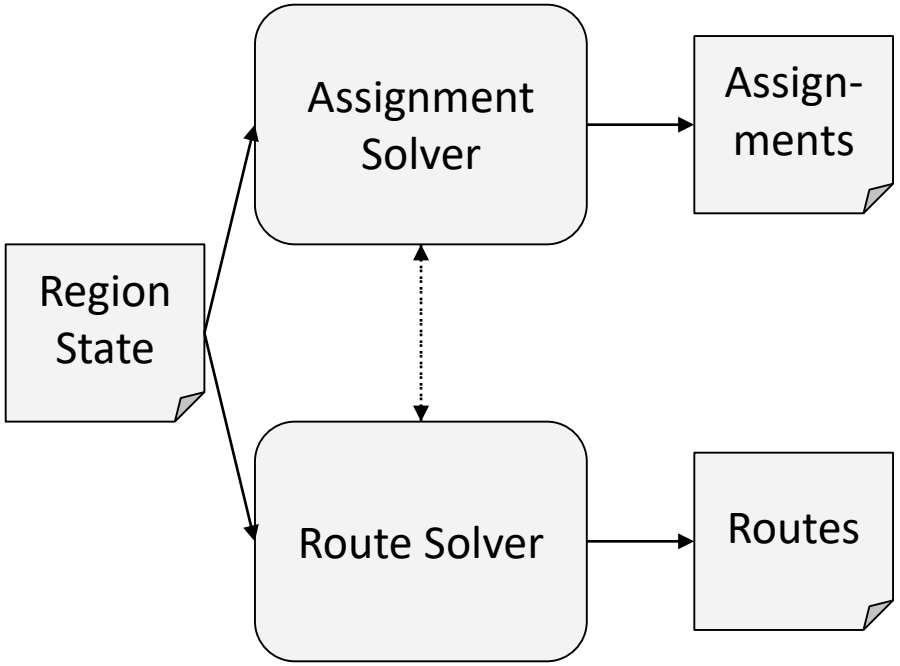
Dynamic Pickup & Delivery for Restaurants

Orders are received dynamically and must be responded to quickly.

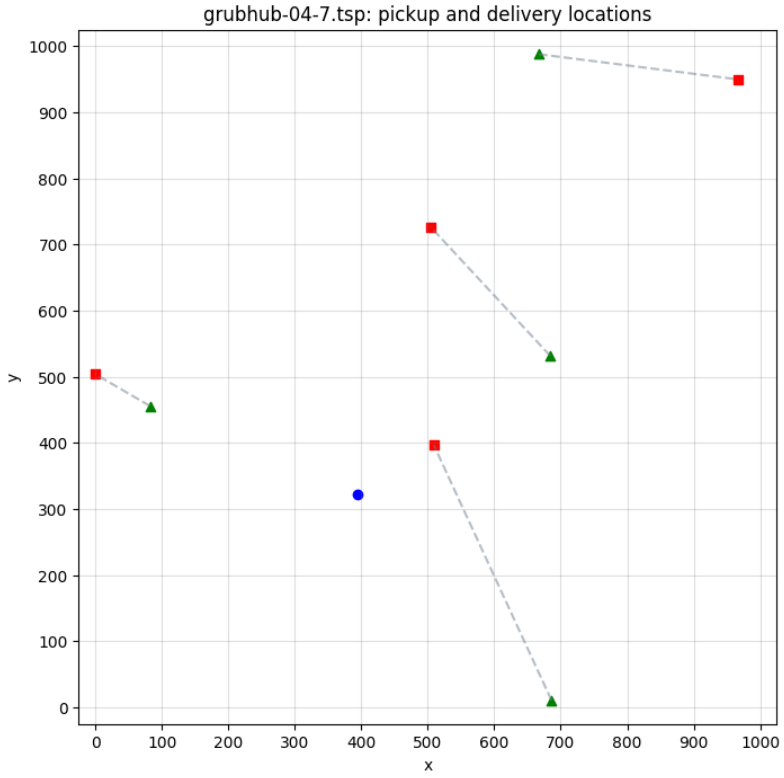
There are hard constraints on plan time.

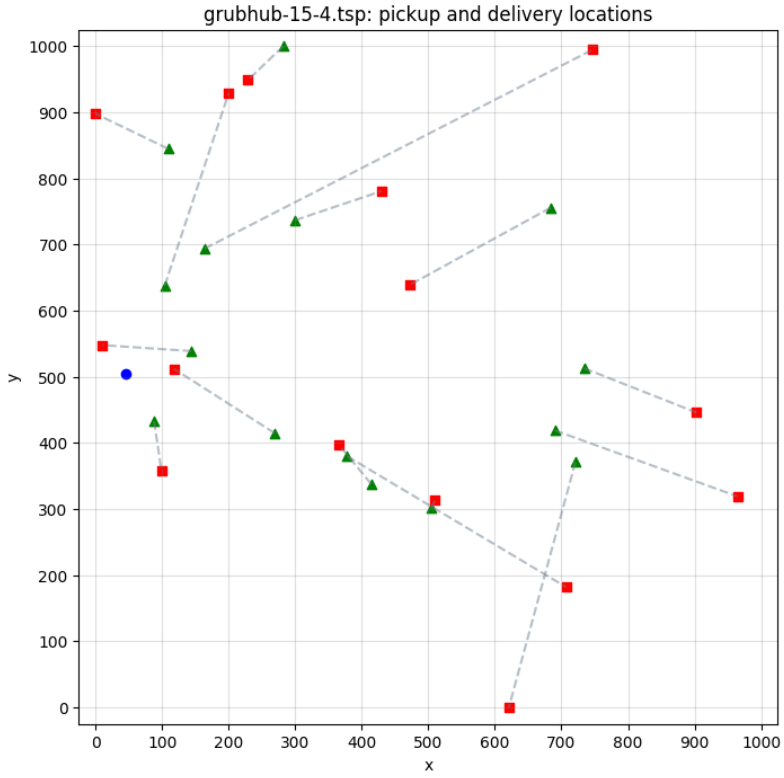
Decomposing assignment and routing and iteratively improving the plan ensures planning can stop when it hits the time budget.

Caveat: The route solving has to be *really fast*, and *has its own time budget*.



Pickup & Delivery: Single Courier Problems





Courier

Pickup

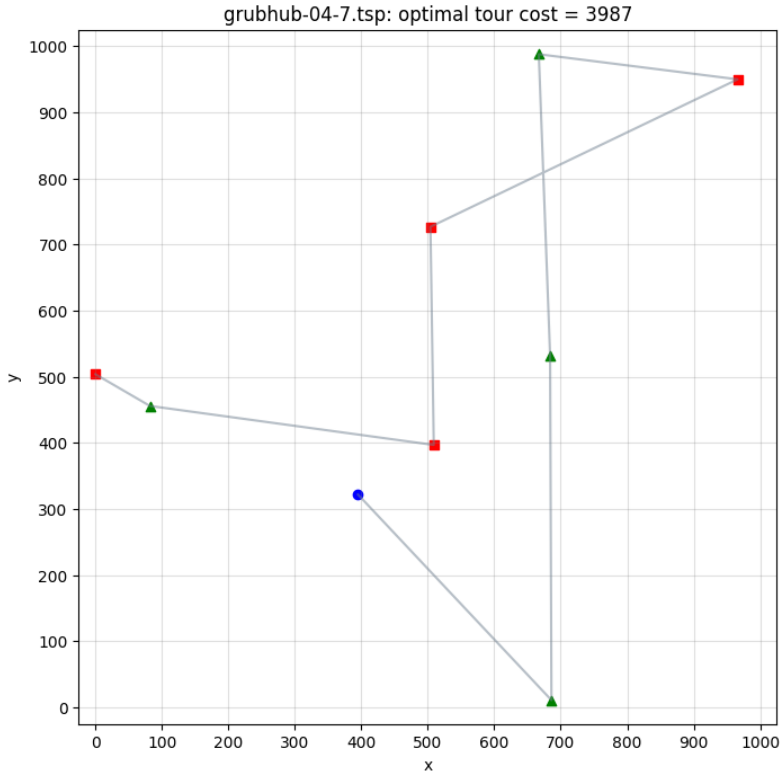
Delivery

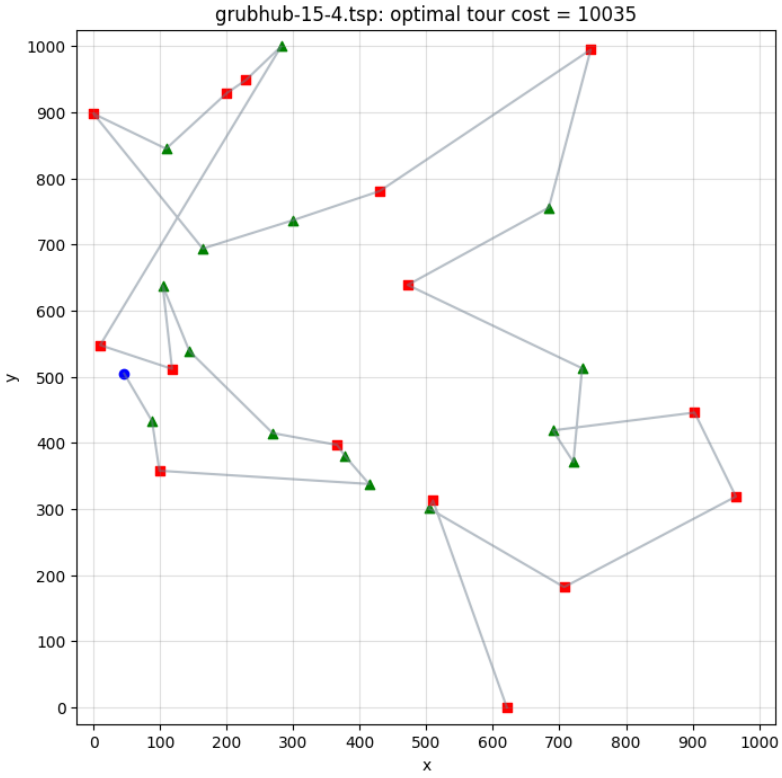
People, Meals, Perishable Goods

Groceries, Packages, Non-Perishable Goods

n

Pickup & Delivery: Single Courier Solutions





Courier

Pickup

Delivery

People, Meals, Perishable Goods

Groceries, Packages, Non-Perishable Goods

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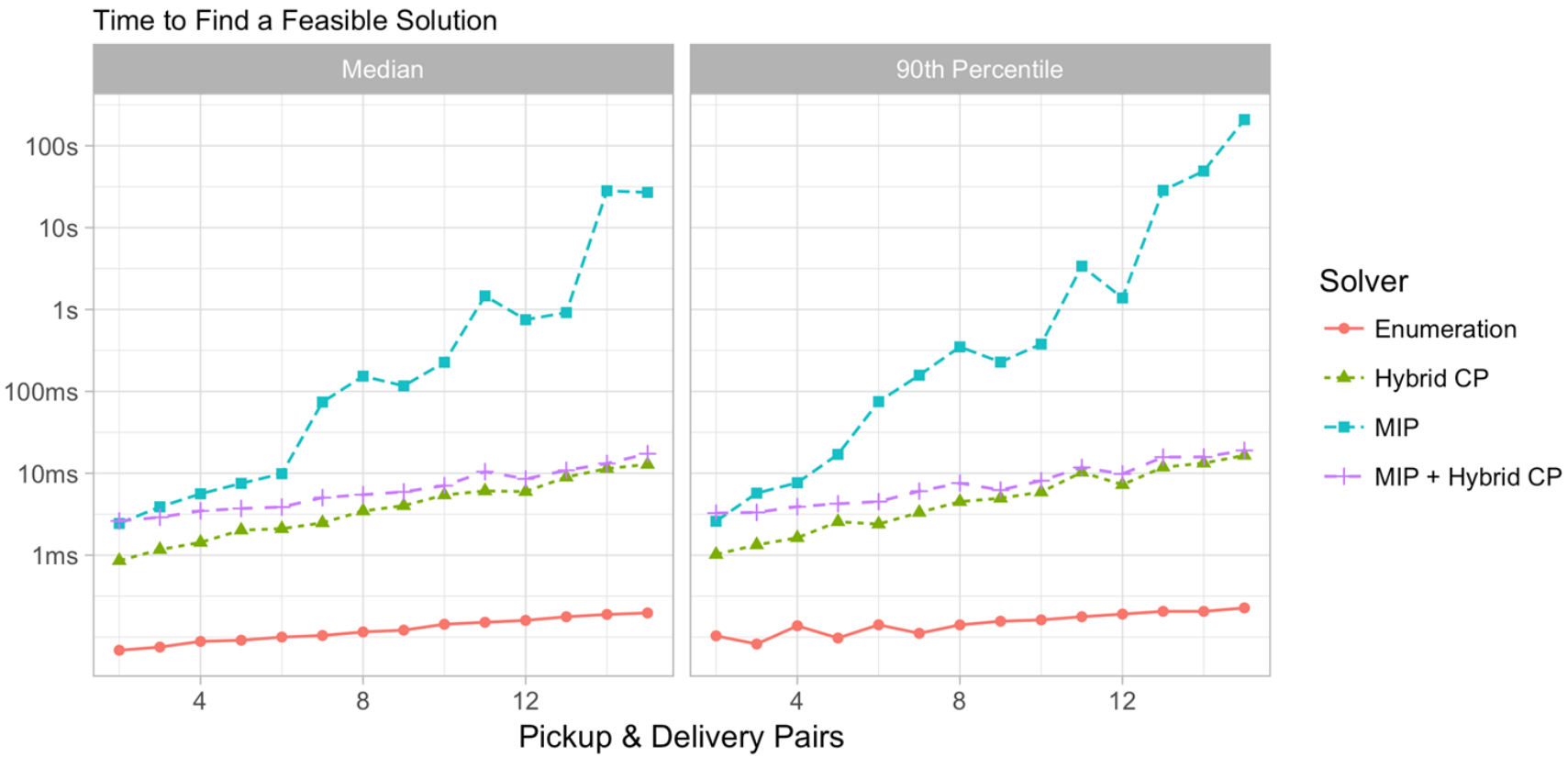
Solution Method Depends on Problem Size & Urgency

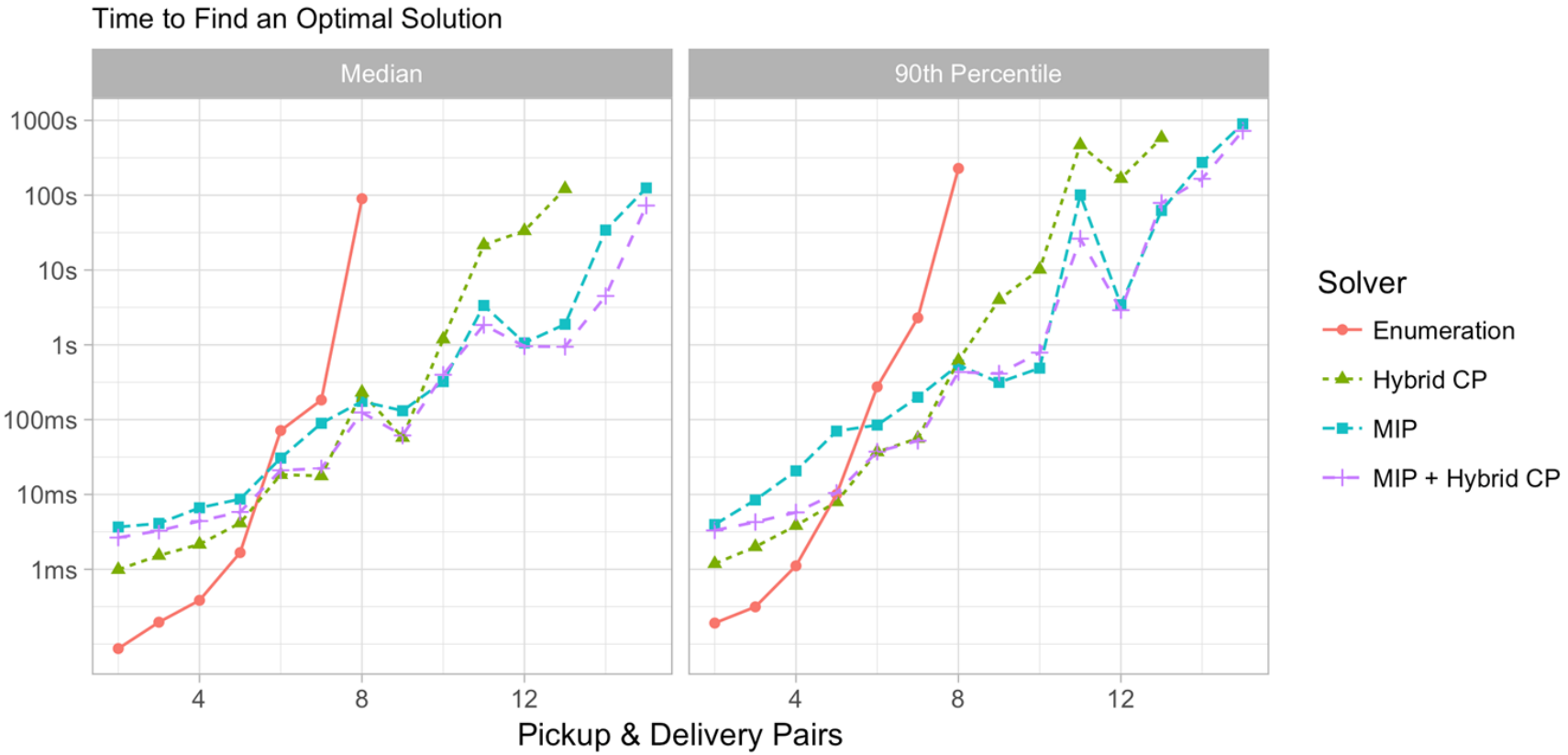
Methods Considered

- Enumeration: Depth-First Search + Greedy Node Ordering + Fathoming
- Hybrid CP: Circuit Constraint + Precedence + Sequential Greedy Branching + AP Reduced Cost Domain Filtering
- MIP: 2-Matching Relaxation + Subtour Elimination + Precedence
- MIP+Hybrid CP: MIP Warm Started with Hybrid CP

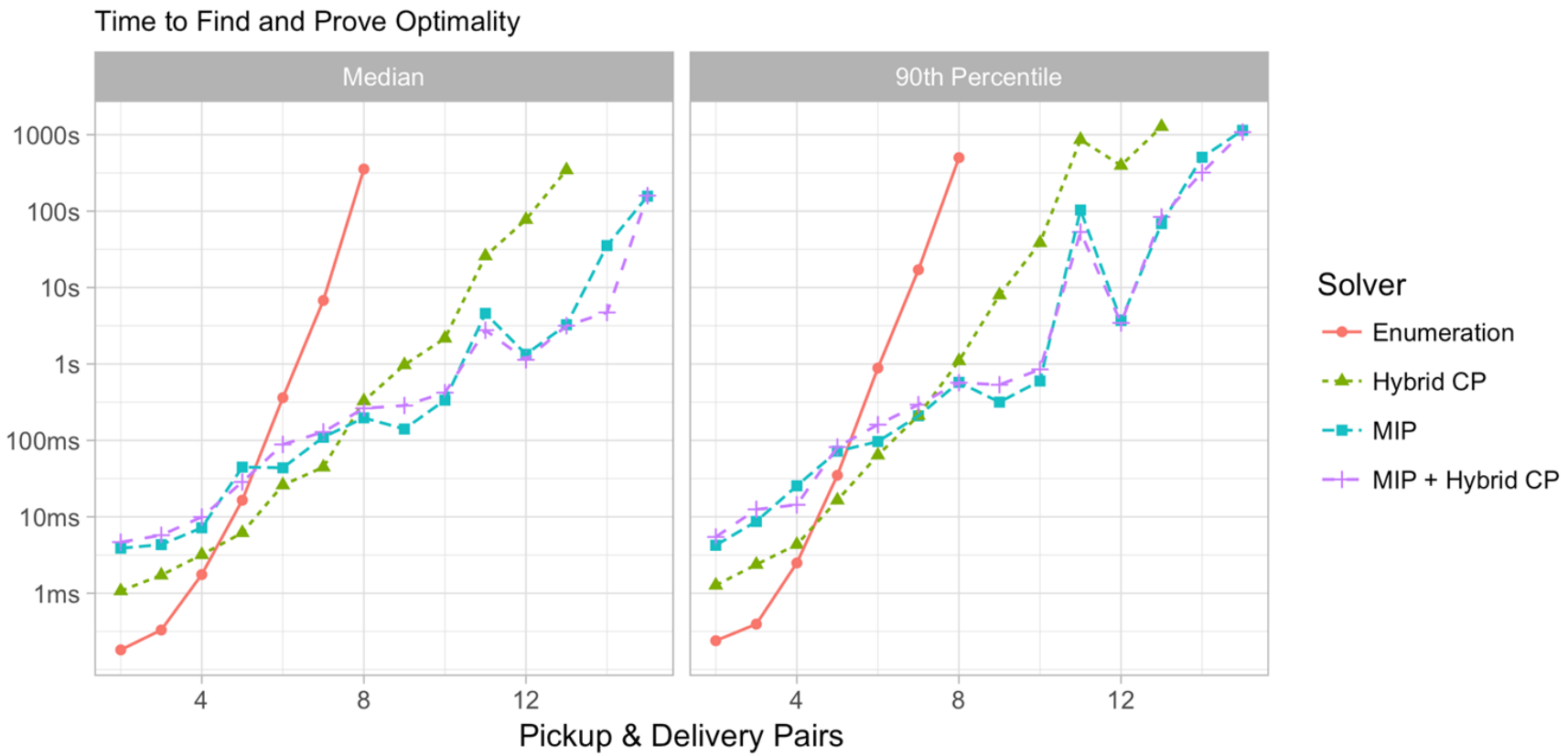
How quickly can I act on new information?



How quickly can I make a really good decision?



How quickly can I be sure I have the best solution?



Solution Methods

When solving hard problems:

1. Formulation matters
2. Exploit problem structure
3. Build your own heuristics
4. Take advantage of the features of the solver
5. Consider multiple optimization approaches

Questions